Thermal and electric oscillation driven by orbit loss in helical systems

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Coupled oscillation of the radial electric field, the heating power, and the plasma temperature in helical systems is analyzed. This oscillation is caused by the influence of the radial electric field on the direct loss of injected fast ions. The damped oscillations of the radial electric field, temperature and fast-ion loss are predicted. Conditions for the occurrence of the mode and the oscillation frequency are discussed.

Recently the important role of the radial electric field on the confinement of toroidal plasmas is widely recognized. This is especially true for the plasmas in the toroidal helical devices such as torsatron/heliotron. This is because the trajectory of the helically trapped particles is strongly influenced by the radial electric field.[1,2] Recent investigation has been focused on understanding the influence of the particle loss on the radial electric field, including the analysis of the self-consistent treatment of the radial electric field and the loss cone loss.[3–5] The majority of these self-consistent analyses for helical plasmas, however, have been limited to the problems of the stationary solutions. The dynamics, which is originated from the mutual influences between the loss cone loss and the electric field, has been studied associated with the H-mode physics in tokamaks.[6] An oscillation of the loss of the fast ions has been reported on Heliotron-E device experimentally by Zushi and co-workers.[7] This phenomena, which we call the Zushi oscillation, casts a motivation for the investigations. This is because the possible stationary operation is considered to be one of the main advantage of helical systems in fusion research.

In this Brief Communication, we report the analytic study on the dynamics of the coupling between the radial electric field, plasma temperature, and loss cone loss in helical plasmas. The loss cone boundary is a functional of the potential profile, so that the heating power is affected by the plasma potential. The heating power dictates the evolution of the plasma temperature. At the same time, the plasma temperature strongly influences the plasma potential. This closes a link between the plasma temperature, the radial electric field and the loss cone loss of injected fast ions. In the following we study the dynamics of this link of processes, showing the existence of the damped oscillations. The analysis is limited to the point model, simplifying the transport problems, in order to show the qualitative nature of the possible thermal and electric oscillations in helical plasmas. In order to keep the simplicity and clarity of the argument, we here assume that the density profile is fixed.

We study the plasma heated by the neutral beam injection (NBI) with the injection energy W_b. The relation of the radial electric field and plasma temperature has been studied in helical systems. For the NBI heated plasmas, a simple formula,

$$E_r \simeq h_i T_i'/e + \delta E, \tag{1}$$

works for the wide range of parameters,[4] where the coefficient h_i is the neoclassical coefficient (close to 3.5). The term δE is the contribution of the direct loss of fast ions. The expression for δE is given as[3,4]

$$\delta E \simeq -\Gamma_b \left(\frac{\partial \Gamma_i}{\partial E_r} \right)^{-1}, \tag{2}$$

where Γ_b is the particle flux of fast ions, and Γ_i is that of the bulk ions. For the parameters of the plasma in present experiments, we have

$$\frac{\partial \Gamma_i}{\partial E_r} = \frac{e D_i n}{T}, \tag{3}$$

where D_i is the neoclassical ion diffusivity, n is the plasma density, and we take $T_i = T_e = T$ for the simplicity. Integrating δE, we have the contribution of the change δE to the potential difference, $\delta\phi$, as

$$-e\delta\phi \simeq \Delta_{\text{layer}} (T/D_i n)_a \Gamma_b, \tag{4}$$

where Δ_{layer} is the thickness of the layer in which δE has a substantial contribution. Noting the relation that $\Gamma_b = P_{\text{loss}}/(4\pi^2 aR W_{\text{loss}})$, where P_{loss} is the loss power associated with the loss cone loss, a and R are minor and major radii, respectively, and W_{loss} is the typical energy of loss ions, we have

$$e\delta\phi \simeq \frac{\Delta_{\text{layer}}}{4\pi^2 aR D_i n} \frac{T}{W_{\text{loss}}} P_{\text{loss}}. \tag{5}$$

Summarizing Eqs. (1) and (5), the potential difference between the center and edge, ϕ_0, is given as

$$e\phi_0 = \left(-h_i T(0) - f_1 T_* \frac{T(a)}{W_{\text{loss}}} \frac{P_{\text{loss}}}{P_{\text{in}}} \right). \tag{6}$$

The negative value of ϕ indicates that E_r is negative, i.e., the static potential is lower at the axis. In Eq. (6), f_1 is a numerical coefficient defined by $a\Delta_{\text{layer}} n / 2\tau_E D_i n(a)$, τ_E is the energy confinement time, n is the average density. The value T_* is the averaged temperature, which would be realized in the absence of the direct loss, given as

$$nT_* = \tau_E P_{\text{in}} / 2\pi^2 a^2 R, \tag{7}$$

and P_{in} is the total injection power.

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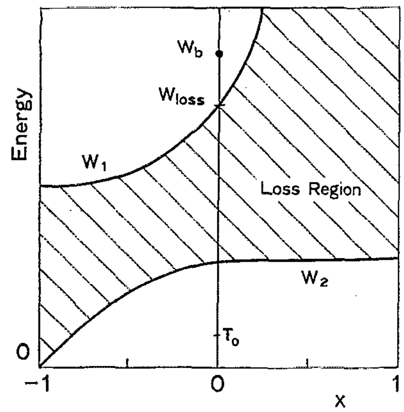

FIG. 1. Loss cone region of the perpendicularly injected fast ions in torsatron/heliotron configuration. The initial energy W_b and the energy at the loss boundaries, W_1 and W_2, are also noted. Expression for W_1 and W_2 are given by Eq. (8). T_0 on the axis shows the thermal level at the plasma center.

The loss cone boundary has been given in literatures.[1,2] Figure 1 illustrates the region of loss cone for the particles injected perpendicular to the magnetic field. In the case of the negative E_r, which we consider here, the loss cone region for the deeply trapped particles exists in the core region[2] for a particular energy range as $W_2 < W < W_1$,

$$W_1 = \frac{e[\phi(a)-\phi(r)]}{\epsilon_h(1-x^2)-\epsilon_t(1+x)} \tag{8a}$$

and

$$W_2 = \frac{e[\phi(a)-\phi(r)]}{\epsilon_h(1-x^2)+\epsilon_t(1-x)}, \tag{8b}$$

where ϵ_h and ϵ_t are the helical and toroidal magnetic ripple at the plasma edge, respectively, and $x = r/a$. Equation (8) is derived for the case that the potential profile is close to the parabolic shape. Monotonic profile of negative radial electric field is observed in CHS torsatron/Heliotron,[8] for which Eq. (8) provides a simple but good approximation. The experimental observation on the potential difference $|\phi(a)-\phi(0)|$ is in the range of $(1\sim3)\times T_i(0)/e$, giving $W_1 \sim (5\text{–}15)\times T_i(0)$ and $W_2 \sim (2.5\text{–}7.5)\times T_i(0)$ at the axis (note that $\epsilon_h \sim 0.3$ and $\epsilon_t \sim 0.1$). The Heliotron-E experiment is done in the range of $W_b \sim (20\text{–}50)T_i(0)$. The case of our interest satisfies the relation $T_i < W_2 < W_1 < W_b$, as is shown in Fig. 1.

We consider the case where the deposition of the fast ions are localized at the center with the specific injection angle of the helically trapped particles. (The extension for the case of the diffused deposition and for the case of the mixed injection angles does not change the result qualitatively.) The boundary for the loss cone is given as $W_{\text{loss}} = W_1(0)$, i.e.,

$$W_{\text{loss}} = -e\phi/(\epsilon_h - \epsilon_t). \tag{9}$$

When the condition $W_{\text{loss}} < W_b$ holds, as is the case of present experiments, the heating by the fast ions takes

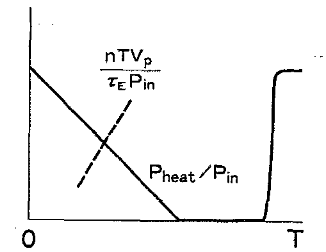

FIG. 2. Heating efficiency, $P_{\text{heat}}/P_{\text{in}}$, and the normalized loss power of the energy transport, $nTV_p/\tau_E P_{\text{in}}$ (dashed line), are plotted as a function of the temperature for the perpendicular injection. The crossing of two lines gives the stationary solution. When the temperature is very high and $W_2(0) > W_b$ holds, then the injected fast ions are confined by the $E\times B$ rotation, and the heating efficiency increases again.

place during the slowing down process from the energy W_b to W_{loss}. Therefore, the power which is effective for the heating, P_{heat}, is given as

$$P_{\text{heat}} = (1 - W_{\text{loss}}/W_b)P_{\text{in}}. \tag{10}$$

Figure 2 illustrates the relation of the heating power as the function of the plasma temperature.

Since the bulk plasma heating occurs through the slowing down process of fast ions, there is a time delay between the power deposition (P_{heat}) and the transferred power to bulk plasma (P). The time evolution of the transferred power per unit volume, P, is simply modeled as

$$\frac{\partial P}{\partial t} = \frac{(P_{\text{heat}}/V_p - P)}{\tau_{\text{heat}}}, \tag{11}$$

where τ_{heat} is the typical slowing down time and V_p is the plasma volume.

The evolution of the plasma temperature is given by the energy transport equation

$$\frac{\partial T}{\partial t} = \frac{(P/n - T)}{\tau_E}. \tag{12}$$

Equations (11) and (12) with Eqs. (6), (9), and (10) constitute the set of basic equations.

We take the normalized form $\hat{T} = T/W_b$, $\hat{P} = \tau_E P/nW_b$, and $\tau = t/\tau_E$. Equations (6) and (9)–(12) are rewritten as

$$\frac{\partial \hat{T}}{\partial \tau} = \hat{P} - \hat{T}, \tag{13a}$$

$$\frac{\partial \hat{P}}{\partial \tau} = \sigma\{[1 - C\hat{T} - C_1\hat{P}_{\text{in}}\hat{T}(a)]\hat{P}_{\text{in}} - \hat{P}\}, \tag{13b}$$

where $\sigma = \tau_E/\tau_{\text{heat}}$, $\hat{P}_{\text{in}} = \tau_E P_{\text{in}}/nV_pW_b$, $\hat{T}(a) = T(a)/W_b$, and C and C_1 are coefficients defined as

$$C = h_i \frac{T(0)}{T} \frac{1}{\epsilon_h - \epsilon_t} \tag{14}$$

and $C_1 = Cf_1/h_i$. For the torsatron/heliotron configuration, such as Heliotron-E and CHS, the term in Eq. (6), which is in proportion to f_1, has only a small contribution. We therefore neglect this term here.

Under the condition of our interest corresponding to the present experiments, which is shown in Fig. 1, Eq. (13) has one fixed point. The stationary solution is given as

$$\hat{T}=\hat{P}=\hat{P}_0\equiv\hat{P}_{\rm in}(1+C\hat{P}_{\rm in})^{-1}. \quad (15)$$

The stability of the trajectory near this fixed point is studied by expanding $\hat{T}$ and $\hat{P}$ near $\hat{P}_0$ as $\hat{T}=\hat{P}_0+x$ and $\hat{P}=\hat{P}_0+y$. The linearized equation is given as

$$\begin{bmatrix}\partial x/\partial\tau\\ \partial y/\partial\tau\end{bmatrix}=\begin{bmatrix}-1 & 1\\ -\sigma C\hat{P}_0 & -\sigma\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}. \quad (16)$$

Writing the time dependence as $(x,y)\sim(x_0,y_0)\exp(\lambda\tau)$, we have

$$\lambda=-(1+\sigma)/2\pm\sqrt{(1-\sigma)^2/4-\sigma C\hat{P}_{\rm in}}. \quad (17)$$

This result shows that the oscillation around the stationary solution is possible if the condition

$$4\sigma C\hat{P}_{\rm in}>(1-\sigma)^2 \quad (18)$$

is satisfied. This relation is more easily satisfied for the case that the slowing down time is close to the energy confinement time, $\sigma\simeq 1$, or the case of large C (i.e., ϵ_t is closer to ϵ_h). Since the real part of the eigenvalue λ is always negative, the oscillation is a damped oscillation. The oscillation is more clearly seen if $|\mathrm{Im}\,\lambda|>|\mathrm{Re}\,\lambda|$, or

$$C\hat{P}_{\rm in}>1. \quad (19)$$

In this case, the oscillation frequency, $|\mathrm{Im}\,\lambda/\tau_E|$, is approximately proportional to $\sqrt{P_{\rm in}/\tau_{\rm heat}}$. If Eq. (19) is not satisfied, the damping rate is comparable to or larger than the oscillation frequency, and the oscillation would not be observed.

From Eq. (18), we see that the oscillation is possible only if τ_E and $\tau_{\rm heat}$ is finite. The oscillation requires the relaxation both in the real space and the velocity space.

The ratio between the amplitude of oscillation of the temperature and that of the heating power can be also obtained. From Eq. (6), we see that the potential difference is $h_iT(0)/T$ times larger than the variation of the average temperature. This is also the case of the heating power. Substituting the eigenvalue Eq. (17) into Eq. (16), we have the ratio between y and x, i.e., y_0/x_0, is given by

$$y_0/x_0=(\sigma-1)/2\pm\sqrt{(1-\sigma)^2/4-\sigma C\hat{P}_{\rm in}}. \quad (20)$$

For the case that the oscillation can be observed, i.e., Eq. (19) holds and $\sigma\simeq 1$, y is larger than x. This implies that the variation of the ion loss is more prominent than the change of the temperature.

In summary, the analytic theory of the thermal and electric oscillation in helical plasma is developed. The existence of the coupled oscillation of $(T, E_r, W_{\rm loss}, P_{\rm heat})$, caused by the coupling of the loss cone loss with the radial electric field, is predicted. This newly predicted coupled oscillation is found to be stable. This suggests that the oscillations appear as the postcursor to the sudden and large-amplitude change of temperature (such as the internal disruption). Equation (20) explains the observation that the oscillation is prominent in the loss of energetic ions. The dependence of the oscillation frequency is derived. If the relations $\tau_{\rm heat}\propto T^{1.5}$ and $T\propto P_{\rm in}^{\mu}$ hold, then the frequency is proportional to $P_{\rm in}^{1/2-3\mu/4}$. For the L-mode plasma, we have $\mu\sim 2/5$. The frequency is higher for higher heating power in the L-mode plasmas.

When heating power is very high so that $W_b<W_2$ holds, or

$$\frac{\tau_E P_{\rm in}}{nV_pW_b}>\frac{1}{C}\frac{\epsilon_h+\epsilon_t}{\epsilon_h-\epsilon_t} \quad (21)$$

the radial electric field would be so high that all the injected ions are trapped by the $E\times B$ rotation, and loss cone disappears for the fast ions. The variety in phenomena would be expected.

The result in this note is compared to experimental observations. For typical plasma parameters of Heliotron-E plasmas, $\langle T\rangle=300$ eV, $\langle n\rangle=3\times10^{19}$ m^{-3}, $Z_{\rm eff}=2$, which are realized for deuterium plasma with $P=1$ MW, relaxation times are given as $\tau_{\rm heat}\sim\tau_E\sim 6$ ms. The parameter $C\hat{P}_{\rm in}$ amounts to about 3, and Eq. (19) is satisfied. These parameters confirm that the oscillation is possible. The oscillation frequency ω is predicted about $\omega=300$/s with the damping rate γ of 170/s. The experiments[7] have shown that the period is around 10 ms and decay time is in the range of 30 ms. The point model in this article overestimates the damping rate (about a factor of 6) and underestimates the oscillation frequency (about a factor of 2–3). This oscillation requires both the time delays in the real and velocity space diffusion. The study including the radial profiles, which is necessary for the quantitative comparison with the Zushi oscillation, is left for future study.

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[1] For a review, K. Miyamoto, Nucl. Fusion **18**, 243 (1978). See, also, H. E. Mynick and W. N. G. Hitchon, *ibid.* **23**, 1053 (1983); R. H. Fowler, J. A. Rome, and J. F. Lyon, Phys. Fluids **28**, 338 (1985).

[2] K. Itoh, S.-I. Itoh, A. Fukuyama, and K. Hanatani, Nucl. Fusion **29**, 1851 (1989); K. Itoh, H. Sanuki, J. Todoroki, T. Kamimura, S.-T. Itoh, A. Fukuyama, and K. Hanatani, Phys. Fluids B **3**, 1294 (1991).

[3] K. Itoh, H. Sanuki, and S.-I. Itoh, Nucl. Fusion **32**, 1047 (1992).

[4] H. Sanuki, K. Itoh, K. Ida, and S.-I. Itoh, J. Phys. Soc. Jpn. **60**, 3698 (1991).

[5] K. Itoh, S.-I. Itoh, H. Sanuki, A. Fukuyama, Comment Plasma Phys. Controlled Fusion **14**, 343 (1992).

[6] S.-I. Itoh, K. Itoh, A. Fukuyama, Y. Miura, and the JFT-2M Group, Phys. Rev. Lett. **67**, 2485 (1991).

[7] H. Zushi, Y. Kurimoto, S. Taniguchi, F. Sano, M. Wakatani, S. Sudo, T. Mizuuchi, K. Hanatani, K. Kondo, H. Okada, K. Nagasaki, K. Yaguchi, and T. Obiki, in *Proceedings of 14th International Conference on Plasma Physics and Controlled Nuclear Fusion Research*, Wurzburg, 1992 (International Atomic Energy Agency, Vienna, 1993), Vol. 2, p. 597.

[8] K. Ida, H. Yamada, H. Iguchi, S. Hidekuma, H. Sanuki, K. Yamazaki, and CHS Group, Phys. Fluids B **3**, 515 (1991).